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# Modelling Cardiac Defibrillation

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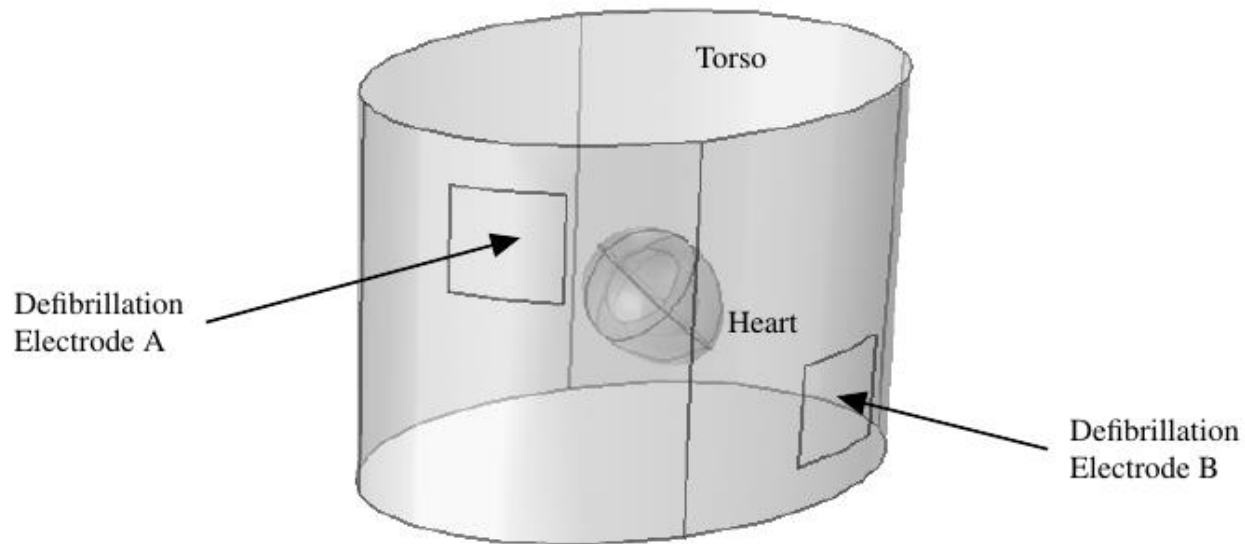
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**EMBC 15 Workshop: Practical Computer Modeling for Medical Device  
Development**

# Model Overview

- In this presentation, we will implement a model of cardiac defibrillation, in which abnormal reentrant activation of the heart is “reset” by an external current applied to electrodes on the wall of the chest. The heart, torso and defibrillating electrodes are represented using the idealised geometry shown below:



# Equations

- Outside the walls of the heart, the electric potential ( $V$ ) is governed by

$$\nabla \cdot (-\sigma_b \nabla V) = 0$$

where  $\sigma_b$  is the electrical conductivity of the torso.

- Within the walls of the heart, extracellular ( $V_e$ ) and intracellular ( $V_i$ ) potentials are defined at every point using the **bidomain** formulation, coupled with modified **Fitzhugh-Nagumo** kinetics for the electrically-active tissue:

$$bC_m \left( \frac{\partial V_e}{\partial t} - \frac{\partial V_i}{\partial t} \right) + \nabla \cdot (-\sigma_e \nabla V_e) = bi_{ion}$$

$$bC_m \left( \frac{\partial V_i}{\partial t} - \frac{\partial V_e}{\partial t} \right) + \nabla \cdot (-\sigma_i \nabla V_i) = -bi_{ion}$$

# Equations (Con't)

$$i_{ion} = c_1 (V_m - a)(V_m - A)(V_m - B) + c_2 u (V_m - B)$$

$$\frac{du}{dt} = e(V_m - du - b)$$

$$V_m = V_i - V_e$$

- $u$  is an auxiliary 'recovery' variable
- $\sigma_e$  and  $\sigma_i$  are the extracellular and intracellular electrical conductivities within the heart
- $\beta$  is the surface to volume ratio
- $C_m$  is cell membrane capacitance per unit area
- $i_{ion}$  is the ionic current per unit cell membrane area
- $A, B, a, b, d, e, c_1$  and  $c_2$  are parameters describing the active electrical activity of the heart.

# Boundary Conditions

- To defibrillate the heart, a rectangular current-pulse of amplitude  $I_d$  and duration  $T_{DUR}$  is applied to defibrillating electrode A at time  $t = T_{ON}$ . Defibrillating electrode B is held at ground.
- All external boundaries of the torso are electrically-insulating, except at the defibrillating electrodes.
- At the boundaries of the heart, the extracellular voltage equals the torso potential and the extracellular current density is continuous.
- For the intracellular potential, the boundaries of the heart are electrically-insulating.

# Parameter Values

Parameter	Value	Parameter	Value
$A$	55 mV	$c_1$	53 nS mV <sup>-2</sup> cm <sup>-2</sup>
$B$	-85 mV	$c_2$	400 μS cm <sup>-2</sup>
$a$	-66.8 mV	$C_m$	1 μF
$b$	-85 mV	$\sigma_e$	0.02 S m <sup>-1</sup>
$d$	140 mV	$\sigma_i$	0.008 S m <sup>-1</sup>
$e$	285.7 V <sup>-1</sup> s <sup>-1</sup>	$\beta$	100 m <sup>-1</sup>
$\sigma_b$	0.2 S m <sup>-1</sup>	$T_{ON}$	840 ms
$I_d$	20 mA	$T_{DUR}$	10 ms

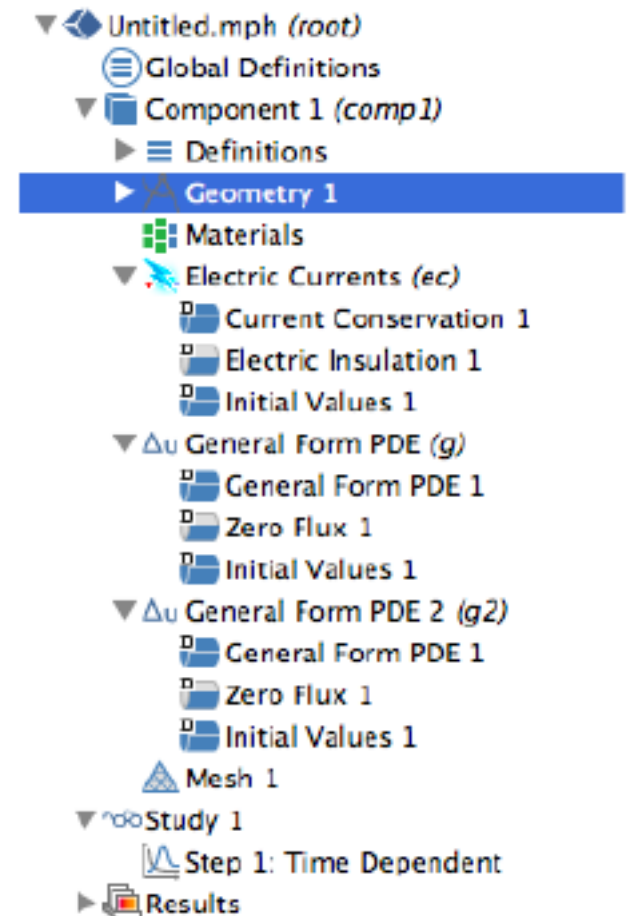
The following slides will provide a step-step guide on how to implement this model using COMSOL Multiphysics...

# Model Wizard

- Open the Model Wizard and select the 3D spatial dimension.
- In the Select Physics panel, choose AC/DC|Electric Currents. Click ``Add``.
- Next, select Mathematics|PDE Interfaces|General Form PDE. Click ``Add``.
- In the Review Physics panel at right, specify  $u$  as the Field name and 2 as the number of dependent variables. In the dependent variables list, enter the names of these variables as  $v_e$  and  $v_i$ . For the dependent variable quantity, specify the units as Electric potential (V), and the source term quantity as Current source ( $A/m^3$ ).

# Model Wizard (Con't)

- Next, select again Mathematics|PDE Interfaces|General Form PDE, and click ``Add''. This will insert a second General Form PDE into the model. In the Review Physics panel, leave the field name as  $u$  and the number of dependent variables as 1. Leave the units of the dependent variable as Dimensionless, but enter the source term units manually as  $1/s$ .
- Click the Study arrow to open the Select Study panel. Select Time Dependent, and click ``Done''. This will exit the Model Wizard, displaying the main COMSOL interface. The model tree will look like that shown at right:



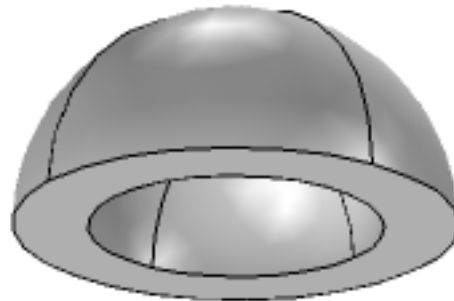


# Geometry

- Select Geometry 1 in the model tree and specify the length unit as mm.
- Right-click Geometry 1 and select Sphere. Specify the radius as 45 mm. Click Build Selected.
- Right-click Geometry 1 and select Sphere again. Specify the radius as 30 mm. Click Build Selected.
- Right-click Geometry 1 and select Boolean Operations|Difference. In the Objects to add field, select the outer sphere (sph1). In the Objects to subtract field, turn on its Active button and select the inner sphere (sph2). To select this, you may need to hide the outer sphere first by clicking the Select and Hide button. This will make the inner sphere visible. Deselect the hide button select the inner sphere. Click the reset hiding button to make both spheres visible. Clicking Build Selected will subtract the inner sphere from the outer.

# Geometry (Con't)

- Next, right-click Geometry 1 and select Block. Specify the width, depth and height as 100 mm, 100 mm and 50 mm respectively. Specify the corner coordinates as (-50 mm, -50 mm, -50 mm), and click Build Selected.
- Right-click Geometry 1 again and select Boolean Operations|Difference. Select the hollowed-out sphere (dif1) as the object to add, and select the block (blk1) as the object to subtract. Click Build Selected. This will result in the hollowed hemispherical 'shell' as shown below:

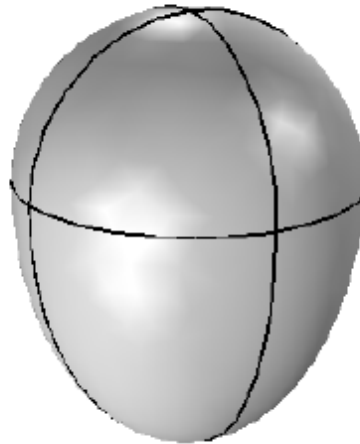


# Geometry (Con't)

- Right-click Geometry 1 again and select More Primitives|Ellipsoid. Specify the a-, b- and c-semiaxes as 30 mm, 30 mm, and 46 mm respectively. Click Build Selected.
- Right-click Geometry 1, selecting again More Primitives|Ellipsoid. This time, specify the a-, b- and c-semiaxes as 45 mm, 45 mm, and 69 mm respectively. Click Build Selected.
- Right-click Geometry 1 and select Block. Specify the width, depth and height to all be 100 mm. Specify the corner of the block to be at (-50 mm, -50 mm, 0 mm). Click Build Selected.
- Right-click Geometry 1 again and select Boolean Operations|Difference. Select the outer ellipsoid (elp1) as the object to add, and select the block (blk2) as the object to subtract. Click Build Selected.

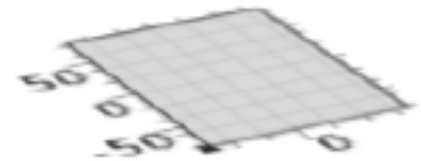
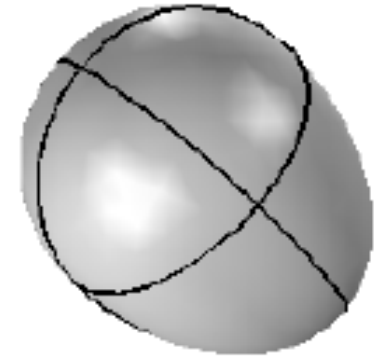
# Geometry (Con't)

- Now right-click Geometry 1 and select Boolean Operations|Difference one more time. Select the outer half-ellipsoid (dif3) as the object to add, and select the inner ellipsoid (elp1) as the object to subtract. Click Build Selected. This will result in a hollowed-out 'heart' object:



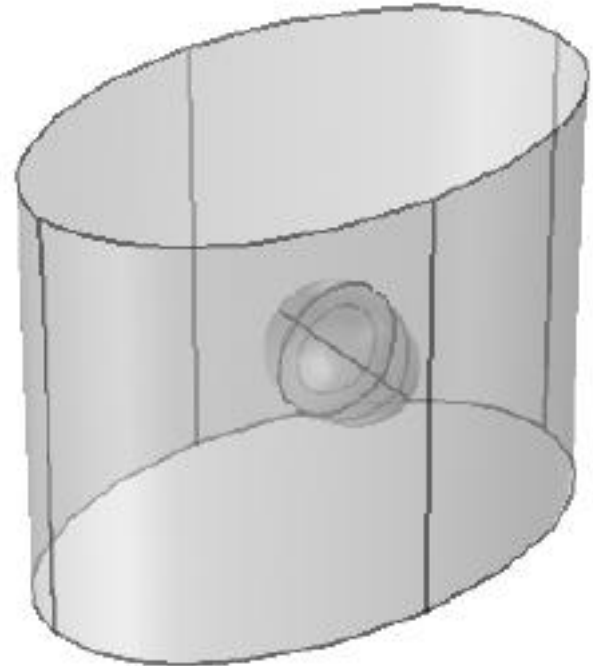
# Geometry (Con't)

- Next, rotate the heart by right-clicking Geometry 1 and selecting Transforms|Rotate. Select both halves of the heart (dif2 and dif4) and specify a rotation angle of  $-45^\circ$ . Specify the axis of rotation as the y-axis. Click Build Selected. This completes the heart geometry.
- We now proceed to specify the torso and defibrillating electrodes. Right-click Geometry 1 and select Work Plane. Define a quick xy-plane (the default setting), and specify the z-coordinate as  $-150$ ,mm. Click Build Selected, followed by the Zoom Extents button in the Graphics window to view the whole geometry with work plane, as shown at right:



# Geometry (Con't)

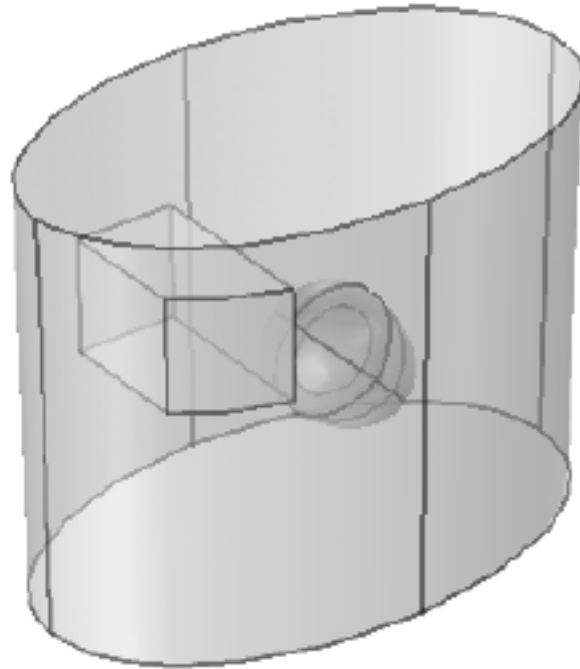
- Now, right-click on the Plane Geometry subnode of Work Plane 1 and select Ellipse. Specify the a- and b-semiaxes as 200 mm and 125 mm respectively. Specify the (xw, yw) position of the centre to be (0 mm, 30 mm) and click Build Selected. Click the Zoom Extents button to see the whole ellipse.
- Next right-click Geometry 1 and select Extrude. By default, the input object of the extrude operation will be the work plane (wp1). Specify an extrude distance from the plane as 300 mm and click Build Selected. This will extrude the ellipse to construct the torso elliptic cylinder. Click Zoom Extents followed by the Transparency button to visualise the heart embedded in the torso as shown at right:



# Geometry (Con't)

- To build the defibrillation electrodes on the torso surface, we specify elongated blocks that intersect with the torso surface. Right-click Geometry 1 and select Block. Specify the width, depth and height to be 70 mm, 150 mm and 80 mm respectively. Specify the centre (not corner) of the block to be at (-125, -30, 75) mm and click Build Selected.
- Next, right-click Geometry 1 and select Transforms|Copy. Select the torso (ext1) and click Build Selected. This creates a copy of the torso. Right-click Geometry 1 and select Boolean Operations|Intersection. Select both the torso (ext1) and the block (blk3) as input objects and click Build Selected. The resulting geometry will look like that shown on the next slide:

# Geometry (Con't)

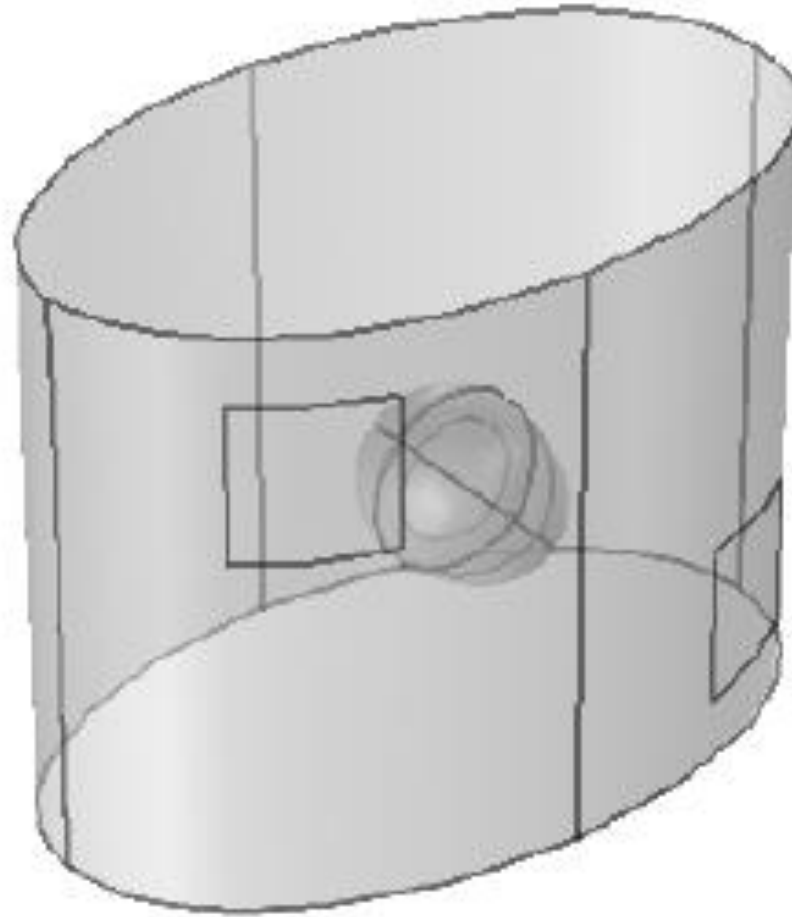




# Geometry (Con't)

- Now we repeat this procedure for the other electrode. Right-click Geometry 1 and select Block. Specify the width, depth and height to be 70 mm, 150 mm and 80 mm respectively. Specify the centre of the block to now be at (125 mm, -30 mm, -75 mm) and click Build Selected.
- As before, right-click Geometry 1 and select Transforms|Copy. Select the torso (copy1) and click Build Selected. Right-click Geometry 1 and select Boolean Operations|Intersection. Select both the torso (copy1) and the block (blk4) as input objects and click Build Selected.
- Finally, right-click Geometry 1 and select Boolean Operations|Union. Specify the two intersection regions (int1 and int2) and the torso (copy2) as the three input objects. Deselect the “Keep interior boundaries” checkbox and click Build Selected. The final geometry obtained is shown on the next slide:

# Geometry (Con't)



# Global Definitions

- Right-click Global Definitions and select Parameters. Enter the following details in the Parameters table of the Settings window:

Name	Expression	Description
A	55 [mV]	Model parameter
B	-85 [mV]	Model parameter
a	-66.8 [mV]	Model parameter
b	-85 [mV]	Model parameter
d	140 [mV]	Model parameter
e	285.7 [1/(V*s)]	Model parameter
c_1	53 [nS/(mV <sup>2</sup> *cm <sup>2</sup> )]	Model parameter
c_2	400 [uS/cm <sup>2</sup> ]	Model parameter
C_m	1 [uF/cm <sup>2</sup> ]	Membrane capacitance
sigma_e	0.02 [S/m]	Extracellular conductivity
sigma_i	0.008 [S/m]	Intracellular conductivity
beta	100 [1/m]	Surface to volume ratio
sigma_b	0.2 [S/m]	Torso bulk conductivity
I	100 [mA]	Defibrillation amplitude
T_on	760 [ms]	Defibrillation onset
T_dur	100 [ms]	Defibrillation duration

- Right-click Global Definitions and select Functions|Rectangle. Specify the lower limit as T\_on and the upper limit as T\_on + T\_dur. In the Smoothing tab, specify the size of the transition zone as T\_dur/10. Leave the function name to its default (rect1).

# Component Definitions

- Right-click the Definitions sub-node of Component 1 and select Component Couplings|Integration. Specify the geometric entity level as 'Boundary' and select boundary 5. This creates an integration operator for integrating expressions over this boundary. Leave the default operator name as intop1.
- Right-click the Definitions sub-node again and select Variables. Leave the geometric entity level to its default as 'Entire model', and enter the following variables in the settings table:

Name	Expression	Description
Area	<code>intop1(1)</code>	Electrode area
J_stim	<code>(I/Area)*rect1(t[1/s])</code>	Current density

# Component Definitions (Con't)

- Again, right-click Definitions and select Variables to define a new variables group. Specify the geometric entity level as 'Domain' and select domains 2 and 3 corresponding to the heart. It may be easier to use the Select Box button in the Graphics window to drag a box around the heart to select these domains. Enter the following in the variables settings table:

Name	Expression
Vm	$V_i - V_e$
i_ion	$c_1 * (V_m - a) * (V_m - A) * (V_m - B) + c_2 * u * (V_m - B)$

# Electric Currents

- Select the Electric Currents node in the model tree. By default this physics is set to hold in all domains of the model (1-4). We wish to override this setting, since this physics should apply only to the torso and not the heart. Select domains 2 and 3 and click the Remove from Selection button to individually remove these domains.
- Expand the Electric Currents node and select the Current Conservation sub-node. In the Settings window, specify the electrical conductivity to be user defined, and enter a value of  $\sigma_b$ . Similarly, specify the relative permittivity to be user defined, and leave the default value of 1.
- Right-click Electric Currents and select Ground. In the Settings window, select boundary 25 (defibrillating electrode B), to set this electrode to ground.

# Electric Currents (Con't)

- Right-click Electric Currents again and select Normal Current Density. Select boundary 5 (defibrillating electrode A) and specify a normal current density of  $J_{stim}$ .
- Right-click Electric Currents again and select Normal Current Density. This time, select all the boundaries of the heart by dragging a select box around it. These correspond to boundaries 6, 7, 9-18, 20-23. For the inward normal current density, enter the expression  
$$\sigma_e * (Vex*nx + Vey*ny + Vex*nz)$$

to specify that the the current density flowing into the torso from the heart is equal to the current density flowing out of the heart's extracellular domain.

# General Form PDE

- Select the General Form PDE node. Again by default, this PDE is set to apply to all domains of the model (1-4). However, since it should be applicable only within the heart, remove domains 1 and 4 using the Remove from Selection button, leaving only domains 2 and 3.
- Select the General Form PDE 1 sub-node of General Form PDE, and enter the following expressions for the conservative flux  $\Gamma$  components for variables  $V_e$  and  $V_i$  respectively:

Component	Expression
x	$-\sigma_e \cdot V_{ex}$
y	$-\sigma_e \cdot V_{ey}$
z	$-\sigma_e \cdot V_{ez}$
x	$-\sigma_i \cdot V_{ix}$
y	$-\sigma_i \cdot V_{iy}$
z	$-\sigma_i \cdot V_{iz}$



# General Form PDE (Con't)

- Enter the following expressions for the source term  $f$  corresponding to variables  $V_e$  and  $V_i$  respectively:

```
beta*i_ion+stim  
-beta*i_ion
```

- Enter the following terms for the damping coefficient matrix  $d_a$ :

```
beta*C_m      -beta*C_m  
-beta*C_m     beta*C_m
```

- Leave the mass coefficient matrix  $e_a$  entries to their default value of 0.

# General Form PDE (Con't)

- Right-click General Form PDE again and select Dirichlet Boundary Condition. Using the select box, drag a selection around the heart to select all heart boundaries. Remove boundary 8 from the selection, which is the internal boundary between the top and bottom parts of the heart. Deselect the 'Prescribed value of  $V_i$ ' checkbox, leaving only the 'Prescribed value of  $V_e$ ' checkbox selected. Enter a value of  $v$  in the  $r_1$  field. This constrains the value of  $V_e$  at the outer and inner boundaries of the heart to equal the torso potential.
- Finally, select the Initial Values 1 sub-node of the General Form PDE node. Leave the initial value of  $V_e$  to its default value of 0. For  $V_i$  however, enter the initial value expression  $-0.085 + 0.12 * (y < 0) * (z > 0.02)$ . This sets  $V_i$  to an initial value of  $0.085 + 0.12 = 0.035$  V for the region of the heart corresponding to  $y < 0$  and  $z > 0.02$  m, representing an electrically-excited state. In all other heart regions,  $V_i$  is initially set to  $-0.085$  V, corresponding to the resting, non-excited state.

# General Form PDE 2

- Select the General Form PDE 2 node. Select domains 1 and 4 and individually remove these using the Remove from Selection button, leaving only domains 2 and 3.
- Select the General Form PDE 1 sub-node of General Form PDE 2, and enter a value of 0 for each of the three components of conservative flux  $\Gamma$ , since there are no spatial derivatives in the equation for  $u$ . For the source term  $f$ , enter the expression  $e * (\nabla m - d * u - b)$  and for the damping and mass coefficients  $d_a$  and  $e_a$ , leave their value as 1 and 0 respectively.
- Finally, select the Initial Values 1 sub-node of the General Form PDE 2 node, and enter the initial value expression  $5 * (x > 0.03)$ . This sets variable  $u$  to an initial value of 5 when  $x > 0.03$  m, corresponding to heart state that is temporarily inexcitable (i.e. refractory), and 0 elsewhere.

# Mesh

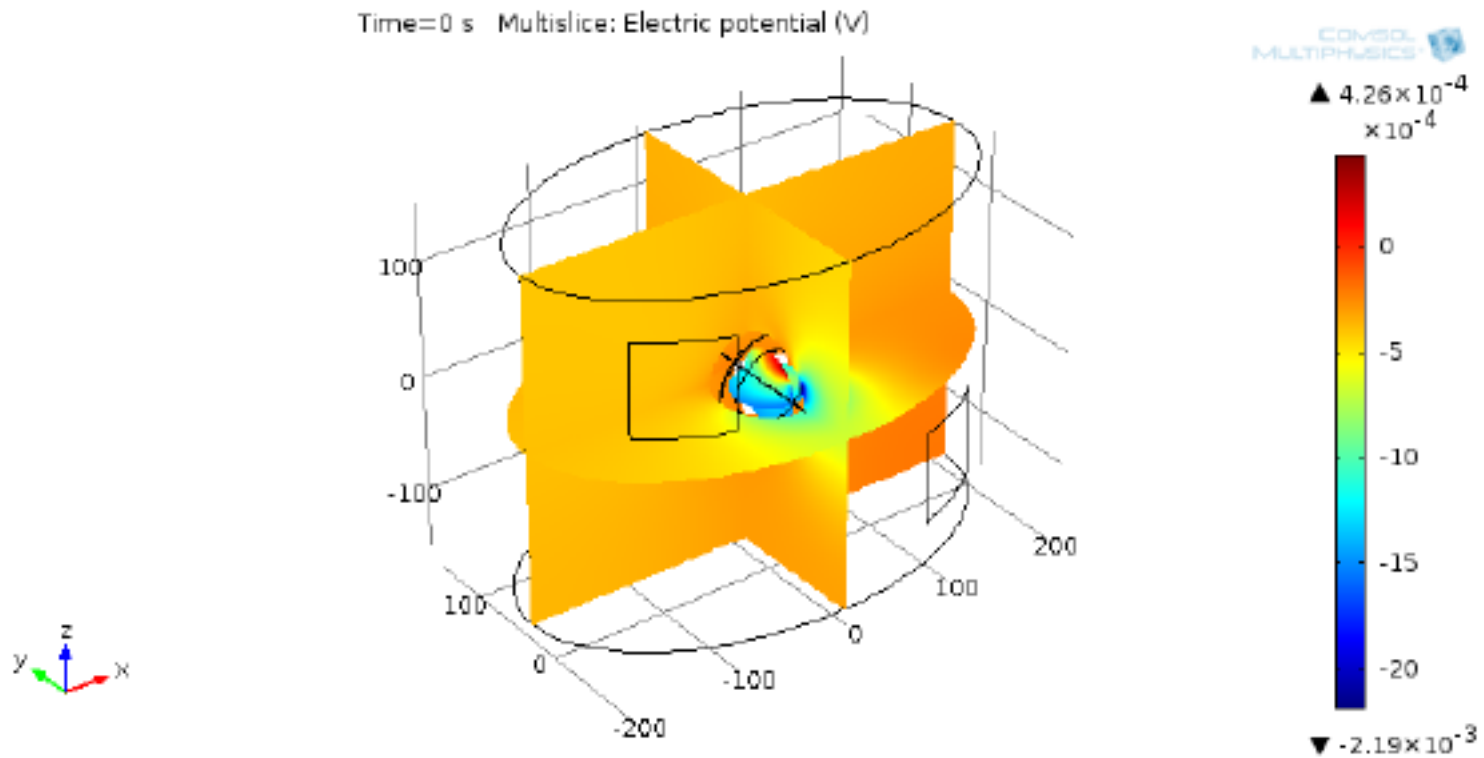
- Right-click Mesh 1 and select Size. In the Settings window, specify 'Domain' as the geometric entity level, and select domains 2 and 3 corresponding to the heart. Under the Predefined Element Size option, select 'Finer' from the dropdown list.
- Right-click Mesh 1 again and select Free Tetrahedral. Leave the default geometric entity level as 'Remaining'. This will mesh the remaining parts of the model with a free tetrahedral mesh.
- Click Build All in the Settings window to build and display the mesh.

# Study

- Select the Step1: Time Dependent sub-node of the Study 1 node. In the Settings window, Click the Range button adjacent to the Times field. Leave the entry method as 'Step' and enter Start, Step and Stop values of 0, 0.001 and 2 respectively. Click Replace. This will create a range of output time values from 0 to 2 s in time steps of 0.001 s.
- Right-click the Study 1 node and select Show Default Solver. Select the Study 1|Solver Configurations|Solution 1|Time-Dependent Solver 1 node. In the Settings window, expand the Advanced tab, and for the 'Singular mass matrix' option, select 'Yes'. Under the Time Stepping tab, select 'Strict' for the Steps taken by solver option.
- To solve the model, right-click Study 1 and select Compute. Select the Progress tab in the Information window to see the solver progress.

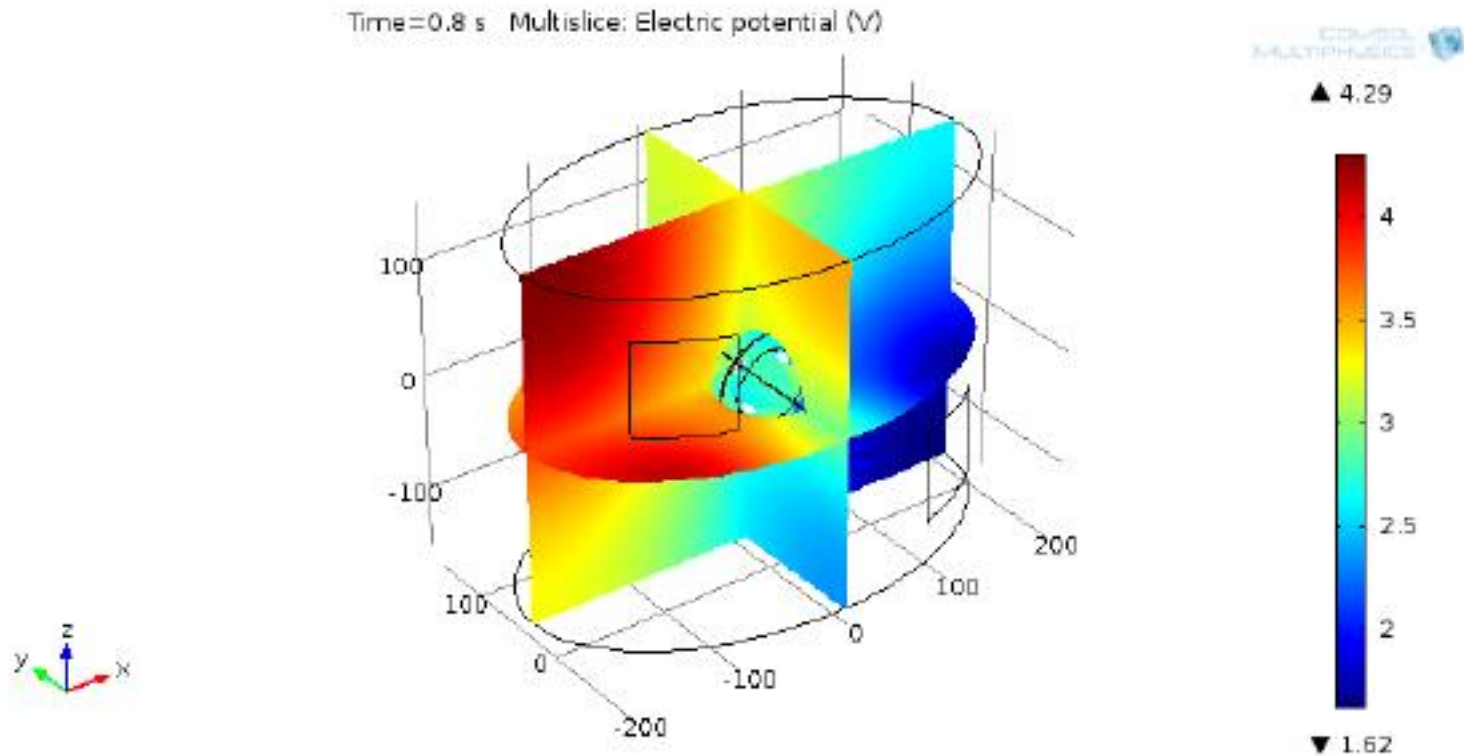
# Results

- When the model has completed solving, the Graphics window will display a default multislice plot of the torso potential (variable  $V$ ) at  $t=0$ :



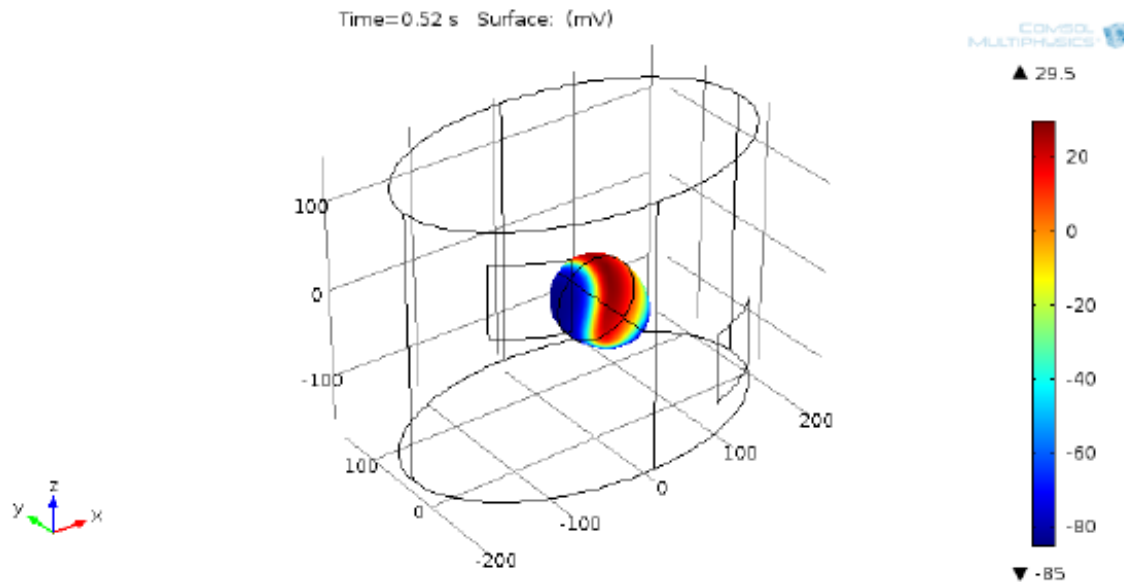
# Results (Con't)

- In the Settings window, select a time of 0.8 s from the Time dropdown list and click the Plot button to display the torso potential at this time, during the defibrillation stimulus:



# Results (Con't)

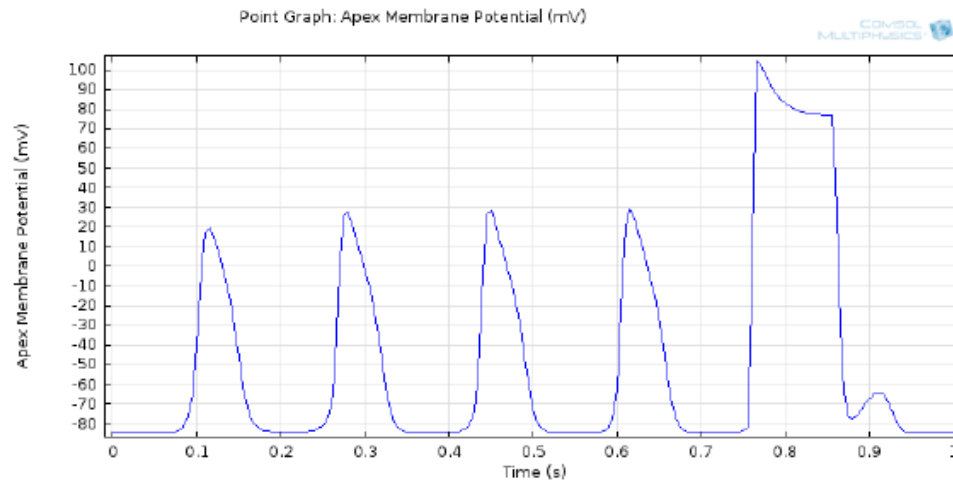
- Right-click Results and select 3D Plot Group. Right-click this new plot group (3D Plot Group 4) and select Surface. In the Settings window, enter  $v_m$  as the expression to plot and select mV as the units from the Unit dropdown list. Left-click the 3D Plot Group 4 node again, and select a time of 0.52 s from the Time dropdown list. Clicking the Plot button will display the membrane potential on the surface of the heart at this time:





# Results (Con't)

- Right-click Results and select 1D Plot Group. Right-click this new plot group (1D Plot Group 5) and select Point Graph. In the Settings window, enter  $v_m$  as the expression to plot and select mV as the units. In the Graphics window, select point 22 corresponding to a point at the apex of the heart. Click the Description checkbox, and type 'Apex Membrane Potential' in the Description field. By default, the x-axis data will be the time values. Clicking the Plot button will display the membrane potential at the apex against time:



# Results (Con't)

- These results show a repetitive sequence of action potentials (electrical excitations) at the surface of the heart prior to 0.7 s, due to re-entrant activation from a wave of excitation travelling continuously around the heart. Following application of an extracellular current between  $t=0.76$  and 0.86 s through the torso surface electrodes, these periodic self-excitations are terminated, indicating successful defibrillation.

